Are the $\Theta^+(1540)$, $\Xi^{--}(1860)$ and $D^{*-}p(3100)$ Pentaquarks or Heptaquarks?

P. Bicudo

Dep. Física and CFIF, Instituto Superior Técnico, Av. Rovisco Pais 1049-001 Lisboa, Portugal

We study the $\Theta^+(1540)$ discovered at SPring-8. We apply Quark Model techniques, that explain with success the repulsive hard core of nucleon-nucleon and kaon-nucleon exotic scattering, and the short range attraction present in pion-nucleon and pion-pion non-exotic scattering. We find a K-N repulsion which excludes the Θ^+ as a $K-N$ s-wave pentaquark. We explore the Θ^+ as a crypto-heptaquark, equivalent to a $K-\pi-N$ borromean boundstate, with positive parity and total isospin I=0. The attraction is provided by the pion-nucleon and pion-kaon interactions. The other candidates to pentaquarks $\Xi^{--}(1860)$, observed at NA49, and $D^{*-}p(3100)$, observed at H1, are also studied as linear molecular heptaquarks.

1 Introduction

The $uudd\bar{s}$ pentaquark $\Theta^+(1540)$ was discovered at LEPS [1] and DIANA [2]. After the Jefferson Lab confirmation [3], it was observed in several different experiences, with a mass of 1540 $\pm$ 10 MeV and a decay width of 15 $\pm$ 15 MeV. Recently the $ddss\bar{u}$ pentaquark $\Xi^{--}(1860)$ was observed at NA49 [4] and the $uudd\bar{c}$ pentaquark $D^{*-}p(3100)$ was observed at H1 [5]. These are extremely exciting states because they may be the first exotic hadron to be discovered, with quantum numbers that cannot be interpreted as a quark and an anti-quark meson or as a three quark baryon. Exotic multiquarks are expected since the early works of Jaffe [6,7], and some years ago Diakonov, Petrov and Polyakov [8] applied skyrmions to a precise prediction of the Θ^+. The $\Xi^{--}(1860)$ and $D^{*-}p(3100)$ belong probably to the same family of exotic flavour pentaquarks.

We start in this talk by reviewing the Quark Model (QM) and the Resonating Group Method (RGM) [9], which are adequate to study states where several quarks overlap. First we apply the RGM to show [10,11,12] that the exotic $N-K$ hard core s-wave interaction is repulsive, excluding the Θ^+ as a bare s-wave pentaquark $uddu\bar{s}$ state or a tightly bound s-wave $N-K$ narrow resonance. However a π could also be present in this system, in which case the binding energy would be of the order of 30 MeV. Moreover this state of seven quarks would have a positive

Table 1: This table summarises the parameters μ, v, α, β (in Fm^{-1}) and scattering lengths a (in Fm) .

channel	μ	v_{th}	α	β	a_{th}	a_{exp}
$K-N_{I=0}$	1.65	0.50	3.2	3.2	-0.14	-0.13 ± 0.04 [12]
$K-N_{I=1}$	1.65	1.75	3.2	3.2	-0.30	-0.31 ± 0.01 [12]
$\pi-N_{I=\frac{1}{2}}$	0.61	-0.73	3.2	11.4	0.25	0.246 ± 0.007 [17]
$\pi-N_{I=\frac{3}{2}}$	0.61	0.36	3.2	3.2	-0.05	-0.127 ± 0.006 [17]
$\pi-K_{I=\frac{1}{2}}$	0.55	-0.97	3.2	10.3	0.35	0.27 ± 0.08 [18]
$\pi-K_{I=\frac{3}{2}}$	0.55	0.49	3.2	3.2	-0.06	-0.13 ± 0.06 [18]

parity, and would have to decay to a p-wave $N-K$ system, which is suppressed by angular momentum, thus explaining the narrow width of the Θ^+. We then put together the $\pi-N$, $\pi-K$ and $N-K$ interactions to show that the Θ^+ is possibly a borromean [13] three body s-wave bound state of a π, a N and a K, with positive parity and total isospin $I=0$. Finally we extend the crypto-heptaquark picture to flavour $SU(4)$ and study the Xi^{--} and $D^{*-}p$ multiquarks.

2 A Quark Model Criterion for Repulsion/attraction

We use a standard Quark Model Hamiltonian. The Resonating Group Method is a convenient method to compute the energy of multiquarks and to study hadronic coupled channels. The RGM was first used by Ribeiro [14] to explain the $N-N$ hard-core repulsion.

We compute the matrix element of the Hamiltonian in an antisymmetrised basis of hadrons,

$$V_{\substack{meson\,A \\ baryon\,B}} = \frac{\langle \phi_B\, \phi_A | (1+P_{AB})[-(V_{13}+V_{23}+V_{14}+V_{24})P_{13}+A_{23}+A_{14}]|\phi_A \phi_B\rangle}{\langle \phi_B\, \phi_A|(1+P_{AB})(1-P_{13})|\phi_A\phi_B\rangle} \ , \qquad (1)$$

where the exchange operator P_{14} produces the states colour-octet x colour-octet, expected in multiquarks, and where A_{23} is the quark-antiquark annihilation potential, crucial to the chiral symmetry of the interaction [15,16].

The exchange overlap results in a separable potential, and we arrive at the criterion for the interaction of ground-state hadrons:
- whenever the two interacting hadrons have a common flavour, the repulsion is increased,
- when the two interacting hadrons have a matching quark and antiquark the attraction is enhanced.

3 Why the Θ^+ cannot be a simple $uudd\bar{s}$ or $K-N$ state

Applying the criterion to the S=1, I=0 pentaquark, arranged in the color singlet clusters $uud-d\bar{s}$ or $ddu-u\bar{s}$ we find repulsion! Indeed we are able to reproduce the repulsive K-N exotic s-wave phase shifts, which have been understood long ago [10,11,12]. Moreover all other $uudd\bar{s}$ systems are even more repulsive or unstable. Because we checked all our only approximations, say using a variational method, and neglecting the meson exchange interactions, we estimate that something even more exotic is probably occuring!

Suppose that a $q-\bar{q}$ pair is added to the system. Then the new system may bind. Moreover the heptaquark had a different parity and therefore it is an independent system (a chiral partner). Here we propose that the Θ^+ is in fact a heptaquark with the strong overlap of a $K-\pi-N$, where the π is bound by the I=1/2 $\pi-K$ and $\pi-N$ attractive interactions.

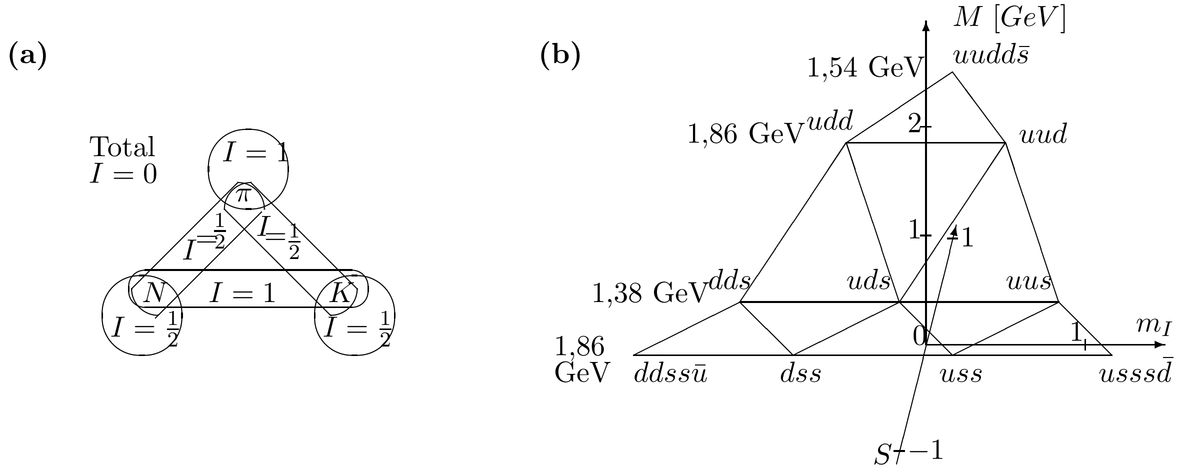

Figure 1: We show in **(a)** the isospin couplings in the Θ^+. In **(b)** we exhibit, in a three dimensional strangeness-flavour-mass plot, the expected masses of the exotic anti-decuplet.

4 the $K-N$, $\pi-K$, $\pi-N$ and $K-\pi-N$ systems

We now investigate the borromean [13] binding of the exotic Θ^+ constituted by a N, K and π triplet. We arrive at the separable potentials for the different two-body potentials [11,16],

$$
\begin{aligned}
V_{K-N} &= \frac{2-\frac{4}{3}\vec{\tau}_A\cdot\vec{\tau}_B}{\frac{5}{4}+\frac{1}{3}\vec{\tau}_A\cdot\vec{\tau}_B}\frac{(M_\Delta-M_N)}{3}\left(\frac{2\sqrt{\pi}}{\alpha}\right)^3 e^{-\frac{{p_\lambda}^2}{2\beta^2}}\int\frac{d^3p'_\lambda}{(2\pi)^3}e^{-\frac{{p'_\lambda}^2}{2\beta^2}} \\
V_{\pi-N} &= \frac{2}{9}(2M_N-M_\Delta)\,\vec{\tau}_A\cdot\vec{\tau}_B\,{\mathcal{N}_\alpha}^{-2}\ , \\
V_{\pi-K} &= \frac{8}{27}(2M_N-M_\Delta)\,\vec{\tau}_A\cdot\vec{\tau}_B\,{\mathcal{N}_\alpha}^{-2}\ ,
\end{aligned}
\tag{2}
$$

where $\vec{\tau}$ are the isospin matrices.

Because the pion is quite light we start by computing the pion energy in an adiabatic K-N system. Our parameter set, tested in 2-body channels, is presented in Table 1. The only favourable flavour combination is shown in Fig. 1 **(a)**. Indeed we get quite a bound pion, but it only binds at very short $K-N$ distances. However when we remove the adiabaticity, by allowing the K and N to move in the pion field, we find that the pion attraction overcomes the $K-N$ repulsion but not yet the the $K-N$ kinetic energy. Other effects may further increase attraction. We are planning to include full three-body Fadeev equations, the coupling to the $K-N$ p-wave channel and the short-range two-pion-exchange-interaction.

5 $SU(4)$ flavour: the $\bar{K}-N-\bar{K}$ and anti-charmed systems

Extending the pentaquark and the molecular heptaquark picture to the full SU(3) anti-decuplet we arrive at the picture shown in Fig. 1 **(b)**, where,
-The $\Xi^{--}(1860)$ cannot be a $ddss\bar{u}$ pentaquark because it would suffer from repulsion.
- Adding a $q-\bar{q}$ pair we arrive at a $I=1/2$ $\bar{K}-N-\bar{K}$ linear molecule where the the $\bar{K}-N$ system has isospin I=1, and it is an attractive system. We find that the $\bar{K}-N-\bar{K}$ molecule is bound, although we are not yet able to arrive at a binding energy of - 60 MeV.
- Then the $I=1/2$ elements of the exotic anti-decuplet are $K-\bar{K}-N$ molecules.
- Only the I=1 elements are pentaquarks, or equivalently overlapping $\bar{K}-N$ systems.

In what concerns anti-charmed pentaquarks like the very recently observed $D^{*-}p$, or anti-bottomed ones, this extends the anti-decuplet to flavour $SU(4)$ or $SU(5)$. Anti-charmed pentaquarks were predicted by many authors, replacing the s by a c. Again the pentaquark $uudd\bar{c}$ is unbound, and we are researching the possible $D(D^*)-\pi-N$ molecular heptaquarks.

6 Conclusion

We conclude that the $\Theta^+1540)$, $\Xi^{--}(1860)$ and $D^{*-}p(3100)$ hadrons very recently discovered cannot really be s-wave pentaquarks.

- We also find that they may be a heptaquark states, with two repelled K and N clusters bound third π cluster.

- More effects need to be included, say exact Fadeev equations, the K-N p-wave coupled channel, and medium range interactions.

- This is a difficult subject with the interplay of many effects. The theoretical models should not just explain the pentaquarks, they should also comprehend other hadrons.

Acknowledgments

Most of the work presented here was done in collaboration with Gonçalo Marques [19].

References

1. T. Nakano *et al.* [LEPS Collaboration], Phys. Rev. Lett. **91**, 012002 (2003) [arXiv:hep-ex/0301020].
2. V. V. Barmin *et al.* [DIANA Collaboration], Phys. Atom. Nucl. **66**, 1715 (2003) [Yad. Fiz. **66**, 1763 (2003)] [arXiv:hep-ex/0304040].
3. S. Stepanyan *et al.* [CLAS Collaboration], Phys. Rev. Lett. **91**, 252001 (2003) [arXiv:hep-ex/0307018].
4. C. Alt *et al.* [NA49 Collaboration], Phys. Rev. Lett. **92**, 042003 (2004) [arXiv:hep-ex/0310014].
5. [H1 Collaboration], arXiv:hep-ex/0403017.
6. R. L. Jaffe, Phys. Rev. D **15** 281 (1977).
7. D. Strottman, Phys. Rev. D **20** 748 (1979).
8. D. Diakonov, V. Petrov and M. V. Polyakov, Z. Phys. A **359** 305 (1997) [arXiv:hep-ph/9703373].
9. J. Wheeler, Phys. Rev. **52**, 1083 (1937); **52**, 1107 (1937).
10. I. Bender, H. G. Dosch, H. J. Pirner and H. G. Kruse, Nucl. Phys. A **414**, 359 (1984).
11. P. Bicudo and J. E. Ribeiro, Z. Phys. C **38**, 453 (1988); P. Bicudo, J. E. Ribeiro and J. Rodrigues, Phys. Rev. C **52**, 2144 (1995).
12. T. Barnes and E. Swanson, Phys. Rev. C **49** 1166 (1994); J. S. Hyslop, R. A. Arndt, L. D. Roper and R. L. Workman, Phys. Rev. D **46** 961 (1992).
13. M. V. Zhukov, B. V. Danilin, D. V. Fedorov, J. M. Bang, I. J. Thompson and J. S. Vaagen, Phys. Rept. **231**, 151 (1993); J. M. Richard, arXiv:nucl-th/0305076.
14. J. E. Ribeiro, Z. Phys. C **5**, 27 (1980).
15. P. J. Bicudo and J. E. Ribeiro, Phys. Rev. D **42**, 1611 (1990); 1625 (1990); 1635 (1990).
16. P. Bicudo, S. Cotanch, F. Llanes-Estrada, P. Maris, J. Ribeiro and A. Szczepaniak, Phys. Rev. D **65**, 076008 (2002) [arXiv:hep-ph/0112015]; P. Bicudo, Phys. Rev. C **67**, 035201 (2003); P. Bicudo, M. Faria, G. Marques and J. Ribeiro, Nucl. Phys. A **735**, 138 (2004) [arXiv:nucl-th/0106071].
17. C. Itzykson and J. B. Zuber, 'Quantum Field Theory," published by Mcgraw-hill, New York, Usa (1980).
18. L. Nemenov, "Lifetime Measurement of $\pi^+\pi^-$ and $\pi^\pm K^\pm$ atoms to test low energy QCD" L23 Letter of Intent for Nuclear and Particle Physics Experiments at the J-PARC (2003).
19. P. Bicudo and G. M. Marques, Phys. Rev. D **69**, 011503 (2004) [arXiv:hep-ph/0308073]; P. Bicudo, arXiv:hep-ph/0403146.